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A NOTE ON PARAMETRIC STABILITY,(U)
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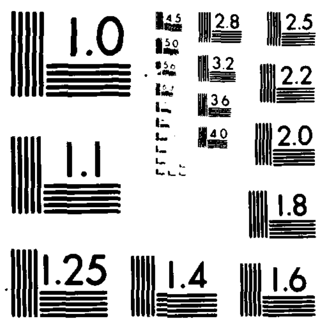

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A NOTE ON PARAMETRIC STABILITY

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ABSTRACT

This note shows how recent developments in the complex variable analysis of multivariable feedback systems can be used to determine stability with respect to a system parameter.

1. Introduction

A feedback system is said to be stable if all of its closed-loop poles are in the left half-plane. The stability of a control system is therefore dependent on its associated parameters. Sometimes in a control system the value of a parameter is uncertain perhaps due to ageing, deterioration, or damage; in other instances it may be desirable, for economic reasons, to change a parameter value. In both these cases a technique which predicts the relative stability of a system with respect to a given parameter would be extremely useful.

A dominant theme in recent research on complex variable techniques for multivariable feedback systems (MacFarlane and Postlethwaite 1977; MacFarlane, Kouvaritakis and Edmunds 1977; Postlethwaite 1978), has been the association of a system with two sets of algebraic functions: characteristic gain functions and characteristic frequency functions. In section 2 characteristic 'parameter' functions are introduced, and used to develop the ideas of 'parametric' root loci and 'parametric' Nyquist loci from which the relative stability of a system, with respect to a single parameter, can be determined. To help in assessing the degree of stability, generalizations of gain and phase margin are given in section 3. The ideas are demonstrated by an example in section 4. In the concluding section tentative proposals and suggestions for future research are made.

2. Characteristic frequency and characteristic parameter functions

The feedback configuration considered is shown in figure 1, where $A(k_2,k_3,\ldots k_q)$, $B(k_2,k_3,\ldots,k_q)$, $C(k_2,k_3,\ldots,k_q)$ and $D(k_2,k_3,\ldots,k_q)$ are state-space matrices which are dependent on (q-1) real, time-invariant parameters and k_1 is a scalar, time-invariant gain parameter common to all the loops.

Figure 1. Feedback configuration for parameter analysis

The closed-loop poles for this configuration are solutions of

$$\det\ [sI_n - S(k)] = 0 \tag{2.1}$$

where

$$S(k) \triangleq A(k_2,\ldots,k_q) - B(k_2,\ldots,k_q)[k_1^{-1}I_m + D(k_2,\ldots,k_q)]^{-1} C(k_2,\ldots,k_q)$$

is the closed-loop frequency matrix (Postlethwaite 1978). If numerical values for all the parameters except one, k_j say, are substituted into equation (2.1) and k_j considered as a complex variable, then the resulting algebraic equation

(which for simplicity of exposition will be regarded as irreducible) defines a pair of algebraic functions (Bliss 1966) $s(k_j)$ and $k_j(s)$. The algebraic function $s(k_j)$ is called the <u>characteristic frequency function</u> with respect to k_j, and the algebraic function $k_j(s)$ is called the <u>characteristic parameter function</u> for k_j. (Note that the characteristic frequency function $s(g)$ and the characteristic gain function $g(s)$, introduced by MacFarlane and Postlethwaite (1977), are equivalent to $s(-k_1^{-1})$ and $-k_1(s)^{-1}$ respectively).

The branches of $s(k_j)$, for k_j real, clearly define the variation of the closed-loop poles with respect to k_j, and as such are termed <u>parametric root loci</u>. Alternatively, the parametric root loci can be viewed as the 0° phase contours of $k_j(s)$ on the Riemann surface domain for $k_j(s)$, known as the <u>frequency surface</u> for k_j.

Dual to the parametric root loci are the <u>parametric Nyquist loci</u> or <u>characteristic parameter loci</u> which are the branches of $k_j(s)$ as s traverses the imaginary axis. Alternatively, the characteristic parameter loci can be viewed as the $\pm 90^{\circ}$ phase contours of $s(k_j)$ on the Riemann surface domain for $s(k_j)$, which will be called the <u>parameter surface</u> for k_j.

If a particular system has a set of nominal parameter values then it is possible from the set of parameter surfaces to determine which, if any, of the parameters are sensitive with respect to stability. To help in such an assessment the following generalizations of gain and phase margin are introduced.

3. Gain and phase margins

The $\pm 90^{o}$ phase contours of $s(k_j)$ on the parameter surface for k_j trace out the boundary between stable and unstable closed-loop poles and therefore we can define parameter gain and phase margins for k_j about a stable operating point $\overset{o}{k}_j$ which give a measure of the relative stability of the system with respect to k_j.

Parameter gain margin. Parameter gain margin is defined with respect to a stable operating point $\overset{o}{k}_j$ as the smallest change in parameter gain about $\overset{o}{k}_j$ needed to drive the system into instability. Let d_i be the shortest distance along the real axis from a stable operating point $\overset{o}{k}_j$ to the stability boundary (characteristic parameter loci) on the ith sheet of the parameter surface for k_j. Then the parameter gain margin is defined as $\min_i \{d_i: i=1,2,...,n\}$

Parameter phase margin. On each of the n sheets of the parameter surface for k_j imagine that an arc is drawn, centre the origin, from a stable operating point $\overset{o}{k}_j$ until it reaches the stability boundary (characteristic parameter loci). Let ϕ_i be the angle subtended at the origin by the corresponding arc on the ith sheet. Then the parameter phase margin is defined as $\min_i\{\phi_i: i=1,2,...,n\}$.

4. Example

In this section an inverted pendulum positioning system (see figure 2) is considered and its stability analysed with respect to one of its parameters, namely the mass of the carriage

Figure 2. Inverted pendulum positioning system

This system has also been used by Kwakernaak and Sivan (1972), Cannon (1967), and Elgerd (1967). The system can be modelled by the following linearized state differential equation (Kwakernaak and Sivan 1972).

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{F}{M} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{g}{L'} & 0 & \frac{g}{L'} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ 0 \end{bmatrix} u(t) \qquad (4.1)$$

where $u(t)$ is a force exerted on the carriage by a small motor; M is the mass of the carriage; F is the friction coefficient associated with the movement of the carriage; and L' is given by

$$L' = \frac{J + mL^2}{mL} \tag{4.2}$$

where m is the mass of the pendulum; L is the distance from the pivot to the centre of gravity of the pendulum; and J is the moment of inertia of the pendulum with respect to the centre of gravity.

The system is stabilizable using state feedback of the form

$$u(t) = -Kx(t) \tag{4.3}$$

and using the numerical values

$$\frac{F}{M} = 1 \ s^{-1}$$

$$\frac{1}{M} = 1 \ kg^{-1} \tag{4.4}$$

$$\frac{g}{L'} = 11.65 \ s^{-2}$$

$$L' = 0.842m$$

it can be found (Kwakernaak and Sivan 1972) that

$$K = [86.81, \ 12.21, \ -118.4, \ -33.44] \tag{4.5}$$

stabilizes the linearized system placing the closed-loop poles at $-4.706 \pm j\,1.382$ and $-1.902 \pm j3.420$.

We will now look at the parameter surface for M to see how variations in the carriage mass, about an operating point of 1kg, affect the stability of the system. The four sheets of the mass surface, characterized by constant phase and magnitude contours of s(M), are shown in figures 3-6, from which the following stability margins are obtained:

parameter (mass) gain margin $=$ 1 kg

parameter (mass) phase margin $=$ 60°

The gain margin of 1kg corresponds to reducing the carriage mass to zero before instability occurs and the phase margin of 60° indicates adequate damping of the closed-loop system.

Sheets 3 and 4 of the mass surface are characterized solely by left half-plane closed-loop poles whereas sheets 1 and 2 have both stable and unstable closed-loop poles separated by the characteristic parameter loci. The crossings of the real mass axes by the characteristic parameter loci determine bounds on the mass for closed-loop stability analogous to the way in which the characteristic gain loci can be used to determine bounds on k_1 (MacFarlane and Postlethwaite 1977); from sheets 1 and 2 we find that the closed-loop system has a 'stable mass interval' of 0 to 2.125 kg.

5. Conclusion

As indicated in this note recent developments in the complex variable analysis of multivariable feedback systems are not only applicable to gain and frequency but any system parameter and frequency. In particular it has been shown how stability with respect to a given parameter can be examined using characteristic parameter loci (generalized Nyquist loci) and characteristic frequency loci (generalized Evans' root loci) viewed on their appropriate Riemann surfaces. To obtain stability results in terms of more than one parameter variation seems to be a much more complicated problem but one with great practical significance. It is felt that such results might

complex variables.

It is also thought that the parameter surfaces may prove to be useful in the design of parameter-dependent controllers for systems in which a particular parameter suffers large variations during normal operation. For example, the controller of an aircraft engine needs to operate satisfactorily over a wide range of altitudes. A possible design scheme could be

(i) to design real constant controllers at a number of altitudes,

(ii) to obtain an altitude dependent controller by "matrix interpolation", and finally

(iii) to analyse the stability of the system over the whole working range using an "altitude surface".

ACKNOWLEDGMENT

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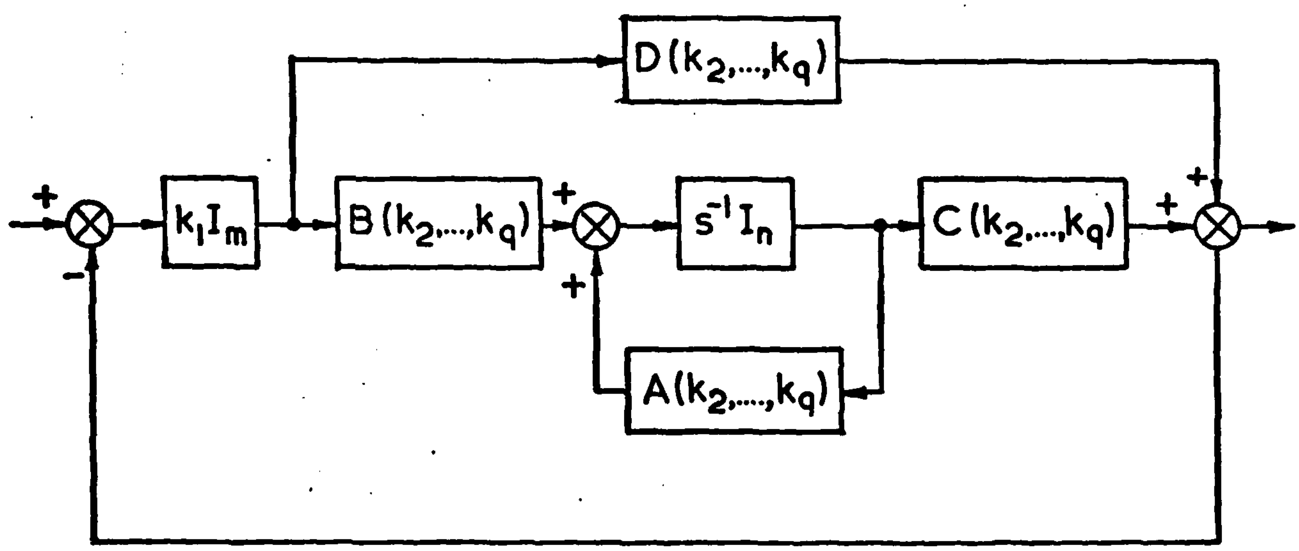

Fig. 1 Feedback configuration for parameter analysis

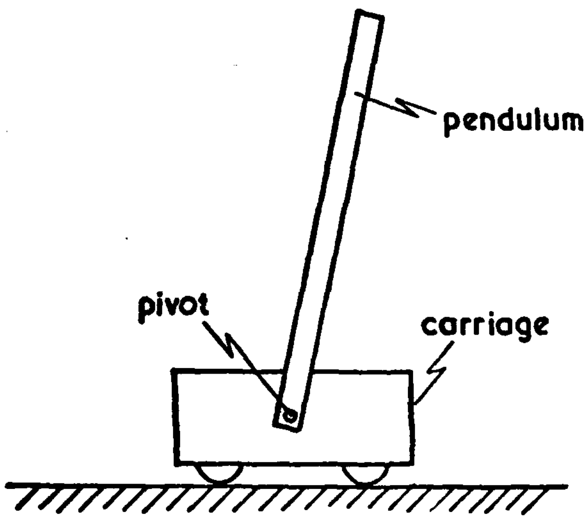

Fig 2 Inverted pendulum positioning system

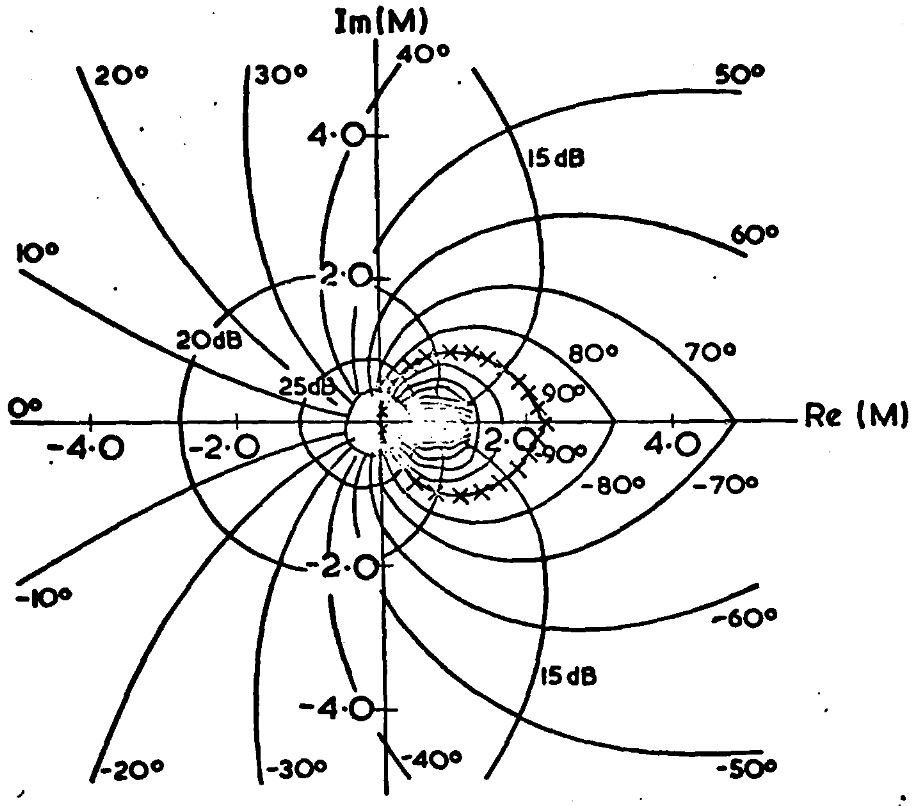

Fig. 3 Sheet 1 of parameter (mass) surface

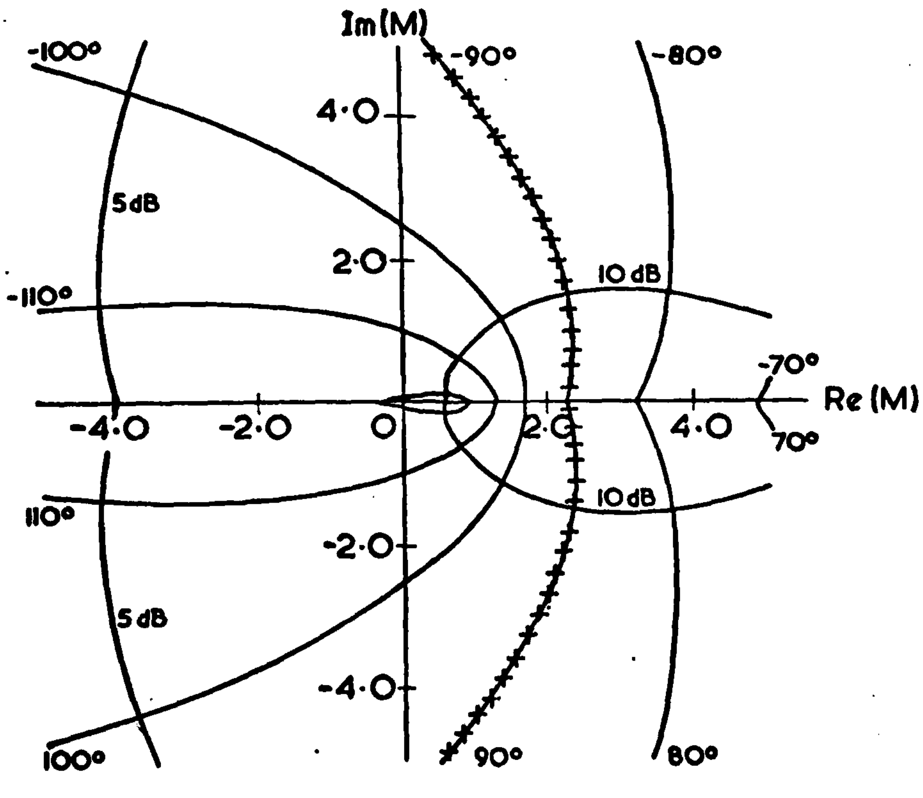

Fig. 4 Sheet 2 of parameter (mass) surface

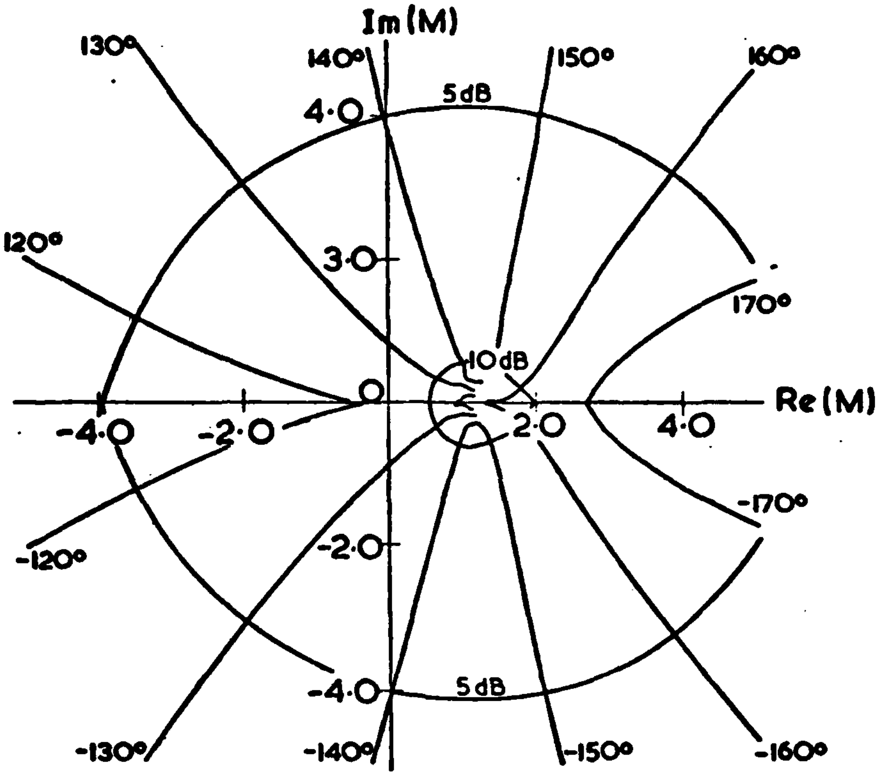

Fig. 5 Sheet 3 of parameter (mass) surface

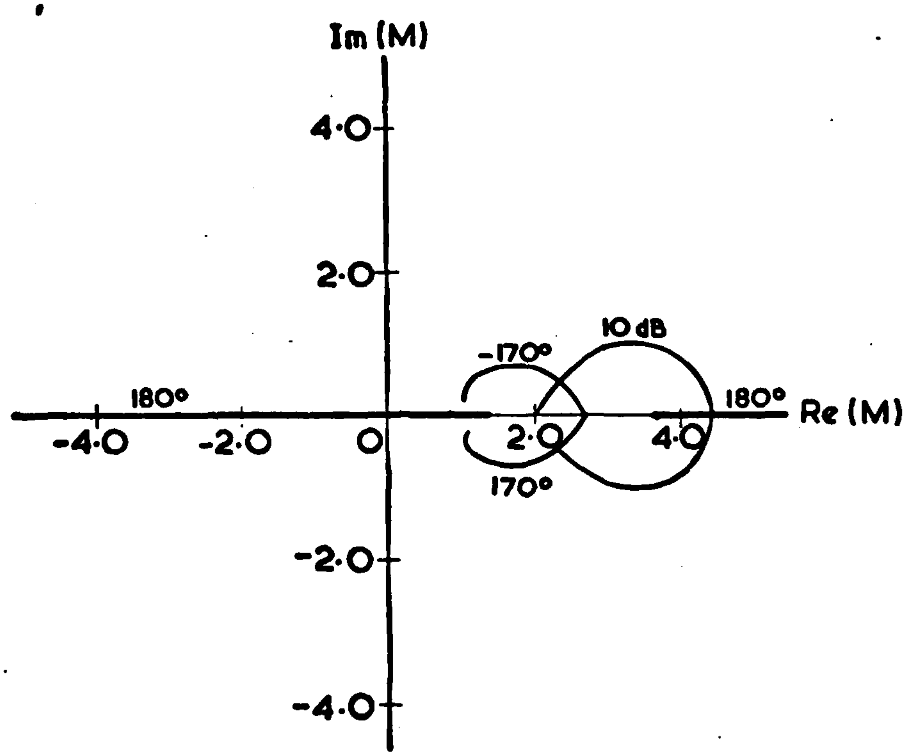

Fig. 6 Sheet 4 of parameter (mass) surface